Theorem: If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$ then for any fixed number $M > 0$ the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML.

Proof: Take an arbitrary $M > 0$. We will begin with the assumption that $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$. By the definition of a limit of a sequence:

$$(*): (\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m \geq n)|a_m - L| < \epsilon$$

If we multiply every entry in $\{a_n\}_{n=1}^{\infty}$ by M then we get $\{Ma_n\}_{n=1}^{\infty}$. Since we know $a_m \in \{a_n\}_{n=1}^{\infty}$ then $Ma_m \in \{Ma_n\}_{n=1}^{\infty}$. This fact will help us later.

Turn attention toward (*). So for all $\epsilon > 0$ there exists an n such that $(\forall m \geq n)|a_m - L| < \epsilon$. $a_m - L$ is either 0, positive or negative. If $a_m - L = 0$ then $Ma_m - ML = 0$ so taking the absolute value $0 = |Ma_m - ML| < \epsilon$.

If $a_m - L > 0$: Reminder it is still the case that $a_m - L < \epsilon$ by the definition of the absolute value. Multiply by M (ϵ is still an arbitrary real number): $Ma_m - ML < \epsilon$.
Taking the absolute value: $|Ma_m - ML| < \epsilon$.

lets take on the case where $a_m - L < 0$. This implies that $|a_m - L| < \epsilon$ is equivalent to $0 > a_m - L > -\epsilon$. Multiply by M: $0 > Ma_m - ML > -\epsilon$. Take the absolute value: $|Ma_m - ML| < \epsilon$

So to reiterate, considering the sequence $\{a_n\}_{n=1}^{\infty}$ for all values of $\epsilon > 0$, there is an n such that for all values of $m \geq n$ where $|a_m - L| < \epsilon$, we found that for any fixed value $M > 0$ such that $|Ma_m - ML| < \epsilon$ and this lines up with the definition of the limit of the sequence: $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML as n tends to ∞. The theorem is proved.